COVERING PROPERTIES OF INVERSE LIMITS, II

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For an inverse system $\{X_\alpha, \pi^\alpha_\beta, \Lambda\}$ and its limit X, let Λ be a directed set its cardinality λ, where $\lambda \geq \omega$, and let π_α be the projection from X into X_α for each $\alpha \in \Lambda$.

Statement (∗). *Let $\mathcal{P}$ be a topological property. Let $\{X_\alpha, \pi^\alpha_\beta, \Lambda\}$ be an inverse system and X its inverse limit with each projection π_α being a pseudo-open map. Suppose that X is λ-paracompact. If each X_α is $\mathcal{P}$, then so is X.*

Aoki [A] and Chiba [C1] proved that the Statement (∗) holds for $\mathcal{P}$ being a covering property such as normality, paracompactness, collectionwise normality, subparacompactness, metacompactness, submetacompactness, subnormality and so on.

Statement (∗∗). *Let $\mathcal{P}$ be a topological property. Let $\{X_\alpha, \pi^\alpha_\beta, \Lambda\}$ be an inverse system and X its inverse limit. Suppose that X is hereditarily λ-paracompact. If each X_α is hereditarily $\mathcal{P}$, then so is X.*

Chiba [C1, C2] also proved that the Statement (∗∗) holds for $\mathcal{P}$ beign the same properties as the above ones.

The purpose of this study is to prove that the Statements (∗) and (∗∗) hold for all main covering properties and all main separation properties. From this point of view, the following three questions have been raised in [C2]:

Question 1. Are Statements (∗) and (∗∗) true for $\mathcal{P}$ being $\delta\theta$-refinability?

Question 2. Are Statements (∗) and (∗∗) true for $\mathcal{P}$ being collectionwise δ-normality?

Question 3. Are Statement (∗) and (∗∗) true for $\mathcal{P}$ being collectionwise subnormality?

Here, we give a partial answer to Question 1. Moreover, we also give affirmative answers to both Questions 2 and 3.

References

[A] Y. Aoki, *Orthocompactness of inverse limits and products*, Tsukuba J. Math. **4** (1980), 241–255.
[C1] K. Chiba, *Normality of inverse limits*, Math. Japonica **35** (1990), 959–970.
[C2] K. Chiba, *Covering properties of inverse limits*, Q & A in Gen. Top. **20** (2002) (to appear).

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