Approximate resolutions and applications to fractal geometry

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For any σ-compact metric space X with a normal sequence $\mathbb{U}$, the Hausdorff dimension $\dim_H^{\mathbb{U}} F$ is defined for each subset F of X, and its properties are investigated. This definition coincides with the usual Hausdorff dimension for subsets of an Euclidean space with a particular normal sequence. In the case that X is a compact metric space, this notion is characterized by a property on an approximate resolution, which is in the sense of S. Mardešić and T. Watanabe. Such an approach using approximate resolutions provides an effective method to construct various examples. In particular, for each $r > 0$, a Cantor set X_r is constructed so that it is realized in the cube $[0,1]^N$ where N is the smallest integer that is greater than or equal to $\frac{\log 3}{\log 2}(r+1)+1$. This generalizes the well-known resut that every compact metric space with covering dimension n is embedded in $[0,1]^{2n+1}$.