ON THE WEIGHT OF COUNTABLY COMPACT GROUPS WHOSE CARDINALITY HAS COUNTABLE COFINALITY

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All spaces considered are Tychonoff. This is a joint work with Irene Castro-Pereira.

In 1980, answering a question from [2], E. K. van Douwen [1] proved that if the Singular Cardinal Hypothesis holds, then, for an infinite cardinal κ of size at least $\mathfrak{c}$, there is no pseudocompact group (countably compact space) of size κ and weight larger than κ if and only if κ has countable cofinality and it is strong limit. He also obtained a characterization for the class of countably compact groups under GCH, made possible by a 'yes' answer under GCH to the following question [1]:

1.5 Question. *If X is an infinite group (or homogeneous space) which is countably compact, is $|X|^\omega = |X|$? Is at least* $\mathrm{cf}(|X|) \neq \omega$*?*

In [3] van Douwen's question was answered, but in the negative. The example had weight smaller than size, thus, the characterization proposed by van Douwen in his question remained untouched. We obtain two new results concerning countably compact groups whose weight is larger than its cardinality. The main one shows that van Douwen's characterization fails without GCH.

References

[1] E. K. van Douwen, *The weight of pseudocompact (homogeneous) space whose cardinality has countable cofinality*, Proc. Amer. Math. Soc. **80** (4), 678–682.

[2] R. E. Hodel, *On the weight of a topological space*, Proc. Amer. Math. Soc. 43 (1974), 470–474.

[3] A. H. Tomita, Two countably compact topological groups: one of size $\aleph_\omega$ and the other of weight $\aleph_\omega$ without non-trivial convergent sequences. Proc. Amer. Math. Soc., to appear.

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