## Extension, Insertion and Selection

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We give a variety of insertion theorems by introducing lower and upper semi-continuity of a map to the Banach space  $C_0(Y)$  for a space Y. As usual, the symbol  $c_0(\lambda)$  is used to denote  $C_0(Y)$  in case Y is a discrete space with cardinality  $\lambda$ .

Theorem 1. For an infinite cardinal, the following conditions on a space X are equivalent:

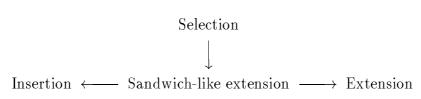
- (1) Every point-finite open cover  $\mathcal{U}$  of X, with  $|\mathcal{U}| \leq \lambda$ , is normal.
- (2) For every two maps  $g, h: X \to c_0(\lambda)$  such that g is upper semi-continuous, h is lower semi-continuous and  $g \le h$ , there exists a continuous map  $f: X \to c_0(\lambda)$  such that  $g \le f \le h$ .

Theorem 2. The following conditions on a space X are equivalent:

- (1) X is paracompact.
- (2) For every space Y and every upper semi-continuous map  $g: X \to C_0(Y)$ , there exists a continuous map  $f: X \to C_0(Y)$  such that  $g \leq f$ .

In Theorem 1 (Theorem 2) the implication  $(1) \Rightarrow (2)$  follows from Kandô-Nedev's (Michael's) selection theorem. The following problem remains open: Is a space X paracompact provided for every space Y and every two maps  $g, h : X \to C_0(Y)$  such that g is upper semi-continuous, h is lower semi-continuous and  $g \leq h$ , there exists a continuous map  $f: X \to C_0(Y)$  with  $g \leq f \leq h$ ?

Sandwich-like extension theorem. Let  $\lambda$  be an infinite cardinal and A a subspace of a space X. Then, A is  $P^{\lambda}$ -embedded in X if and only if for every two maps  $g,h:A\to c_0(\lambda)$  such that g is upper semi-continuous, h is lower semi-continuous and  $g\leq h$ , there exists a continuous map  $f:X\to c_0(\lambda)$  such that  $g\leq f|_A\leq h$ .



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