TOPOLOGICAL SEQUENCE ENTROPY OF MONOTONE MAPS ON ONE-DIMENSIONAL CONTINUA

筑波大学　数学系　知念直紹　(NAOTSUGU CHINEN)

Let f be a continuous map from a continuum X to itself. T. N. T. Goodman introduced in 1974 the notion of topological sequence entropy as an extension of the concept to topological entropy. Let f be a continuous map from a compact metric space X to itself and $\mathbf{A}, \mathbf{B}$ finite open covers of X. Denote $\{f^{-m}(A)|A \in \mathbf{A}\}$ by $f^{-m}(\mathbf{A})$ for each positive integer m, $\mathbf{A} \vee \mathbf{B} = \{A \cap B|A \in \mathbf{A}, B \in \mathbf{B}\}$ and $N(\mathbf{A})$ denotes the minimal possible cardinality of a subcover chosen from $\mathbf{A}$. Let $S = \{s_i|i = 1, 2, \dots\}$ be an increasing unbounded sequence of positive integers. We define $h_S(f, \mathbf{A}) = \limsup_{n\to\infty} \frac{1}{n} \log N(\bigvee_{i=1}^{n-1} f^{-s_i}(\mathbf{A}))$. And we define the *topological sequence entropy of* f (*respect to the sequence* S) as $h_S(f) = \sup\{h_S(f, \mathbf{A})|\mathbf{A}$ is a finite open cover of $X\}$. If $s_i = i$ for each i, then $h_S(f)$ is equal to the standard topological entropy $h(f)$ of f. And set $h_\infty(f) = \sup_S h_S(f)$. If X is a compact interval (by Franzová and Smítal in 1991) or the circle (by Hric in 2000), it was proved that f is chaotic in the sense of Li-Yoke if and only if $h_\infty(f) > 0$.

For an open cover $\mathbf{A}$ of X, we set $\mathrm{Bd}(\mathbf{A}) = \bigcup\{\mathrm{Bd}(A)|A \in \mathbf{A}\}$, where $\mathrm{Bd}(Y)$ denotes the boundary of Y in X. A continuum X is said to be *regular* if for each $\varepsilon > 0$, there exists a finite open cover $\mathbf{A}$ of X with $\mathrm{mesh}\mathbf{A} < \varepsilon$ such that $\mathrm{Bd}(\mathbf{A})$ is finite. A locally connected continuum is said to be a *dendrite* if it contains no simple closed curve. It is known that all dendrites and all graphs are regular. Seidler proved in 1990 that if f is a homeomorphism from a regular continuum X into itself, then $h(f) = 0$. A continuous map $f : X \to X$ is said to be *monotone* if $f^{-1}(Y)$ is connected for each connected subset Y of $f(X)$. Kolyada and Snoha showed in 1996 that if X is either a compact interval or the circle, then $h_\infty(f) = 0$ for all monotone map $f : X \to X$. And Efremova and Markhrova in 2001 considered some class D of dendrites and showed that the topological entropy $h(f)$ of f is equal to zero for all monotone map f from $X \in D$ to itself.

Denote $D_{f,n} = \{x \in X|\mathrm{Card}(f^{-n}(x)) \leq 1\}$ and $D_f = \bigcap_{n=1}^{\infty} D_{f,n}$. A continuum X is said to be *regular for* f if for each $\varepsilon > 0$ there exists a finite open cover $\mathbf{A}$ of X with $\mathrm{mesh}\mathbf{A} < \varepsilon$ such that $\mathrm{Bd}(A)$ is finite contained in D_f for each $A \in \mathbf{A}$. I will prove the following result which is an extension of the aboves: Let f be a monotone map from X into itself. If X is regular for f, then $h_\infty(f) = 0$. This shows that if X is either a dendrite or a graph and f is a monotone map from X into itself, then $h_\infty(f) = 0$.

2000 *Mathematics Subject Classification.* Primary 37B40, 37E10; Secondary 28D05, 54H20.
Key words and phrases. Topological sequence entropy ; monotone map ; regular continuum ; dendrite ; graph.